On the Individual Optimality of Economic Integration*

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March 2011

Abstract

Which countries find it optimal to form an economic union? We emphasize the risk-sharing benefits of economic integration. We consider an endowment world economy model, where international financial markets are incomplete and contracts not enforceable. A union solves both frictions among member countries. We uncover conditions on initial incomes and net foreign assets of potential union members such that forming a union is welfare-improving over standing alone in the world economy. Consistently with evidence on economic integration, unions in our model occur (i) relatively infrequently, and (ii) emerge more likely among homogeneous countries, and (iii) rich countries.

Keywords: Incomplete markets, endogenous borrowing constraints, risk sharing, economic integration.

JEL Codes: F15, F34, F36, F41.

*We thank Martin Gervais, Sílvia Gonçalves, Per Krusell, Mark Wright, our discussants Steve Ambler and Harris Dellas, and seminar attendants at McGill, Atlanta Fed, Queen's University, Washington University in St.Louis/St.Louis Fed, Université de Montréal, University of Windsor, 2010 CMSG conference at Western Ontario, 7th Vienna Macroeconomics Conference at EIEF in Rome, 2010 Midwest Macro Meetings in East Lansing, 2009 CEA meetings in Toronto, 2009 Portuguese Economic Journal meetings in Madeira, and 2009 SED meetings in Istanbul for helpful comments. Castro acknowledges financial support from SSHRC.

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1 Introduction

Which countries find it optimal to form an economic union? Our aim is to try to understand the patterns of economic integration that we observe in the real world. We emphasize a particular motivation for economic integration: improving risk sharing. We view economic unions as small-scale arrangements, comprised of a small number of countries, where partners are better able to cope with the frictions that limit risk-sharing in the world economy. We ask which countries would rather be part of this type of economic union than stand alone in the world economy, and compare the configuration of successful unions predicted by our theory with those we see in the data.

We consider an initial situation in which countries are sitting in the world economy with very limited possibilities to sharing idiosyncratic endowment risk. Risk sharing is limited by two frictions. First, markets are incomplete since countries may only trade a non-contingent bond. Second, international lending contracts are not legally enforceable. At any time, a country may choose to repudiate its foreign debt. The sanction for doing so is the permanent exclusion from future trade in world markets. Our world economy model is a variant of Clarida (1990) and Huggett (1993), featuring self-enforcing borrowing limits along the lines of Kehoe and Levine (1993), Kocherlakota (1996), and Alvarez and Jermann (2000). Versions of this setup have been studied previously in different contexts by Zhang (1997) and Krueger and Perri (2006).[1]

We then consider the possibility that a pair of countries selected at random from the world economy is suddenly offered the possibility of forming an economic union. A union, by assumption, is an arrangement which solves both the market incompleteness and the lack of enforcement problems among member countries. The union as a whole, however, still faces these frictions when trading in world markets. Since the endowment risk facing union members cannot be fully diversified away, they still have an interest in trading with the rest of the world.

The key trade-off our model emphasizes about union formation, from the perspective of each individual country, is the following. There are two benefits from economic integration. First, forming a union improves risk-sharing opportunities among member countries. Second, a union allows for poor partners to use the rich partners' credit lines. The latter is a benefit for poor partners only. There are also two costs of economic integration. First, borrowing limits become tighter, since defaulting on international debt becomes less costly for union partners. This happens

[1]See Ábrahám and Cárceles-Poveda (2010) and Bai and Zhang (2010) for variants with capital accumulation. See also Castro (2005) for a variant with capital accumulation and endogenous but ad-hoc borrowing constraints.

because union partners may still share risk upon default. Second, since poor partners may benefit from the rich partners' credit limit inside the union, this generates a cost for the rich: rich partners will find themselves more often borrowing-constrained in a union compared to standing alone in the world economy.

Our model generates not only benefits, but also costs of economic integration. In addition, our model also generates disagreement about union formation, and the disagreement is the largest when the partners are more heterogeneous. These two ingredients provide a potential explanation for three seemingly puzzling empirical observations on economic integration: (i) deep economic integration is relatively rare, and when it does take place it tends to feature (ii) relatively homogeneous partners, and (iii) relatively richer partners. In other words, we do not tend to see many North-South arrangements; they are mostly North-North, and to a lesser extent South-South. Our paper provides empirical evidence documenting these regularities.

These observations are puzzling because, under a very broad set of circumstances, economic theory would imply that economic integration should happen often, particularly among heterogenous partners. For example, this would be the case for capital market integration in the neoclassical growth model, or goods market integration in either the Heckscher-Ohlin or the Ricardian models of trade.[2]

Our framework provides a very parsimonious explanation for these puzzling observations. Economic unions may not be formed if either the costs of economic integration are too large, or if there is disagreement among partners. Unions are unlikely to be formed among heterogeneous partners, since poor partners impose a cost on the rich. Finally, unions are also more likely to be formed among relatively rich partners because this lowers the likelihood of either country being borrowing-constrained in the future, and thus the source of disagreement.

This paper is related to a vast literature that has attempted to estimate the welfare gains from full international risk-sharing. This literature includes papers such as Cole and Obstfeld (1991),

[2]Union formation in intra-industry trade models, emphasizing scale economies and a taste for variety, have been analyzed in a static setting by Krugman (1991), Frankel, Stein, and Wei (1995), Frankel (1997) and Baier and Bergstrand (2004). This type of model emphasizes size as a determinant of union formation: the larger and the more similar the partners' market sizes, the larger the gains from goods market integration. Larger unions profit more from scale economies, and size homogeneity lowers the losses from trade diversion. While Baier and Bergstrand (2004) find empirical support for these implications, our data also suggests that, *beyond market size*, the level and the dispersion in partner wealth matters for economic integration. Differently from this literature, our paper focuses on heterogeneity in per capita incomes and net foreign assets over GDP.

Backus, Kehoe, and Kydland (1992), Obstfeld (1994b,a), van Wincoop (1994, 1999), Mendoza (1995), Tesar (1995), Lewis (2000), and Athanasoulis and van Wincoop (2000). The typical exercise computes the average gain across countries of going from financial market autarky to complete markets, and entirely eliminating idiosyncratic country risk. Although the range of estimated welfare gains is large, the gains are still positive in nearly all the papers. The sole exception is Devereux and Smith (1994), who like this paper also model costs of sharing risk. In their case, sharing risk lowers precautionary saving, which lowers output growth and might lower welfare. We emphasize instead the tightening of credit constraints, and the costs generated by poor union partners.

Our paper differs from this literature in several dimensions. First, beyond the magnitude of the welfare gains, we are mostly interested on their distribution across countries. Even if the average gains might be high, they can be very oddly distributed. If some countries actually experience a loss, as it is often the case in our model, risk sharing arrangements may not take place at all. This may explain the observed lack of international risk diversification, even in the presence of possibly large average welfare gains. Moreover, the main prediction of our model can be tested against the evidence, namely that feasible risk-sharing arrangements should occur among homogeneous and rich countries.

Second, our paper considers financial market integration as it typically takes place in the real world. That is, as voluntary arrangements among small sets of countries. Financially integrated countries are still unable to share risk with the rest of the world. Further, in our paper countries may save and self-insure in the absence of complete markets, whereas most of the literature abstracts from this feature. Our paper computes welfare gains from international risk-sharing that take these important features into account.

A recent paper that has also looked at potential risk sharing arrangements within small sets of countries is Imbs and Mauro (2008). Using actual data on the variance-covariance matrix of cross-country output growth, they uncover the number and configuration of countries that offer the best risk-sharing potential. Like in the rest of the international risk-sharing literature, they focus on going from autarky to complete markets, and do not feature neither costs of economic integration, nor a role for disagreement among partners. Their main finding is that most diversification gains are achieved in arrangements featuring a small (up to seven) number of countries, and in arrangements between highly volatile countries. As Imbs and Mauro (2008) recognize, a natural question is why we do not observe more arrangements of this type. They argue that this could be because unions might be particularly costly to sustain among volatile countries, since these also tend to have poor contract enforcement institutions. While our framework abstracts from cross-country differences in output volatility, it does provide an explicit, alternative reason for why small-size arrangements may not be feasible, even in the face of large aggregate gains.

The paper is organized as follows. Section 2 presents some evidence about union formation. Section 3 presents the model of the world economy. Section 4 characterizes the union. Section 5 presents the results. Section 6 discusses the European Union experience. Section 7 concludes. Appendix A provides some details about the data. Appendices B and C describe the decentralization of the union's allocation and the numerical algorithm, respectively.

2 Empirical Evidence

We start by providing some empirical evidence on the role of wealth levels and wealth inequality for union formation. By wealth we mean both income (y) and net foreign assets (b), both variables being potentially relevant according to our formal model. Our approach is to run a probit-gravity regression to test whether wealth levels contribute positively, and wealth inequality negatively, for the probability of union formation. Our regression specification is a straightforward adaptation of those commonly used in the empirical trade literature to test predictions over bilateral trade flows (see Frankel and Romer (1999), Frankel and Rose (2002)), similar to Baier and Bergstrand (2004). We consider:

$$\text{Prob}\left\{\text{Union}_{ij} = 1 | X_{ij}\right\} = \Phi(X'_{ij}\boldsymbol{\beta})$$

with

$$\begin{aligned}
X'_{ij}\boldsymbol{\beta} &= \alpha_1 + \alpha_2 \ln(\text{dist})_{ij} + \alpha_3 \text{adj}_{ij} \\
&+ \left(\alpha_4 + \alpha_4^a \text{adj}_{ij}\right) \ln(\text{pop}_i \times \text{pop}_j) + \left(\alpha_5 + \alpha_5^a \text{adj}_{ij}\right) \left|\ln \frac{\text{pop}_i}{\text{pop}_j}\right| \\
&+ \left(\theta_1 + \theta_1^a \text{adj}_{ij}\right) \ln(y_i + y_j) + \left(\theta_2 + \theta_2^a \text{adj}_{ij}\right) \left|\ln \frac{y_i}{y_j}\right| \\
&+ \left(\gamma_1 + \gamma_1^a \text{adj}_{ij}\right) \left(\frac{b_i + b_j}{y_i + y_j}\right) + \left(\gamma_2 + \gamma_2^a \text{adj}_{ij}\right) \left|\frac{b_i}{y_i} - \frac{b_j}{y_j}\right|.
\end{aligned}$$

The dependent variable is a dummy which gets the value of 1 if a union is formed between countries i and j, and 0 otherwise. The regressors in the first two lines of the regression equation concern factors deemed to be important for union formation but absent from our theoretical framework. The last two lines concern wealth levels and wealth heterogeneity, the key determinants in our theory.

We begin with the former set of regressors. We include two geographical factors commonly used in the gravity regression literature, the distance between the main economic centers of countries i and j (dist_{ij}), and a dummy variable capturing whether countries i and j share a common border (adj_{ij}). We also include overall size and a measure of heterogeneity in size, as potential determinants of union formation, where size is measured by population (pop_i). In particular, Baier and Bergstrand (2004) have found scale effects to be important for union formation, consistent with the predictions of a class of intra-industry trade models. In the last two lines, we include the overall income level of the country pair (i, j), a measure of the inequality in incomes between the two countries, and similarly for net foreign assets over income. We make the contribution of wealth levels and wealth inequality for union formation contingent upon whether countries share a border, and similarly for size. Our specification finds a parallel in Frankel and Romer (1999).

To implement our regression analysis, we combine a variety of data sets. From version 6.3 of the Penn World Tables (Heston, Summers, and Aten, 2009) we obtain our measure of income (real GDP per capita) and population. We obtain net foreign asset positions from Lane and Milesi-Ferretti (2007). We consider real GDP and nominal net foreign assets over nominal GDP averaged over five years (2000-2004) as our regressors, to prevent high frequency variation in these variables from affecting our results.

Our geographical data comes from Frankel and Rose (2002), and our union dummy is obtained from a comprehensive data set assembled by Baier and Bergstrand (2009). Based primarily on information from the World Trade Organization, this data set provides information on which countries are engaged in any kind of regional trade arrangement in any given year. The regional trade arrangements range from Preferential Trade Arrangements, to Free Trade Areas like NAFTA, to Economic Unions like the European Union. For reasons that will become apparent when we model unions in Section 4, we restrict our empirical definition of unions only to those arrangements characterized by a sufficiently deep level of economic integration. In particular, we do not consider Free Trade Areas like NAFTA as a union. This is because members of Free Trade Areas may set independent tariff policies vis-a-vis non-members, making it in our view inappropriate to think about them as a block. Our most comprehensive empirical definition of unions includes Customs Unions (no trade barriers between members, common barriers vis-a-vis non-members), Common Markets (custom unions featuring free capital and labor mobility between members), and Economic Unions (common markets featuring harmonization of economic policy, namely fiscal and monetary). We also report regression results for stricter empirical definitions of an economic union, the results being generally robust across them. Appendix A lists existing unions, ordered by depth of integration.

We focus on a single cross-section of 136 countries in the year 2004. The year is the most recent one in the Baier and Bergstrand (2009) data set, and the number of countries is the maximum given the available data in 2004. We then consider all possible country pairings from this set. We assign the value of 1 to the union dummy if a particular country pair was part of a union in 2004, and 0 otherwise. Given the available geographical data, we end up with 6629 country pairings.

We report in Table 1 our estimated average marginal effects, evaluated at either value for the common border dummy.

As expected, our results support a negative effect of distance on the probability of union formation. Regarding scale, the results are not fully consistent with Baier and Bergstrand (2004), in the sense that scale does help union formation, but only for sufficiently deep arrangements, and only conditional on countries not sharing a common border. Otherwise scale has either no significant effect, or is actually detrimental for union formation. Just like Baier and Bergstrand (2004), however, we do find that inequality in scale is generally detrimental to union formation.

We now turn to the variables which are more relevant for us. The evidence supports the view that the larger the partners' combined incomes, the higher the probability of union formation among non-adjacent countries. Income inequality is always clearly detrimental to union formation, and similarly for inequality in net foreign assets over GDP, although with lower statistical significance. The combined level of net foreign assets over GDP tends instead to be detrimental for union formation, except for customs unions with shared borders.[3]

We take these results to support the broad view that, even when controlling for geographical factors and scale effects, wealth levels contribute positively, and wealth inequality contributes negatively to union formation.

Some simple scatter plots help illustrate our basic empirical findings. The left panel of Figure 1 shows the income levels of all country pairs in the sample together with the 45 degree line. The right panel is restricted to those country pairs in a custom union or deeper arrangement. The blue

[3]The variables NFA/GDP and NFA/GDP Inequality capture the level and inequality effects, respectively, of net foreign assets on union formation which are not already captured by the variables Income and Income Inequality. Namely differences in net foreign assets which are proportional to output are captured by the latter variables.

Table 1: Wealth, inequality, and union formation

Marginal Effects on the Probability of Union Formation				
Definition of Union: at least...		...Customs Union	...Common Market	...Economic Union
Distance	adj=0	−0.039 (0.000)	−0.025 (0.000)	−0.014 (0.000)
	adj=1	−0.039 (0.000)	−0.022 (0.000)	−0.019 (0.000)
Population Size	adj=0	−0.002 (0.002)	0.004 (0.000)	0.002 (0.000)
	adj=1	−0.001 (0.640)	−0.001 (0.506)	−0.003 (0.259)
Population Inequality	adj=0	−0.005 (0.000)	−0.002 (0.110)	−0.004 (0.000)
	adj=1	−0.001 (0.743)	−0.004 (0.172)	−0.003 (0.284)
Income	adj=0	0.023 (0.000)	0.016 (0.000)	0.004 (0.010)
	adj=1	−0.001 (0.844)	−0.001 (0.827)	−0.002 (0.699)
Income Inequality	adj=0	−0.025 (0.000)	−0.043 (0.000)	−0.023 (0.000)
	adj=1	−0.028 (0.013)	−0.015 (0.047)	−0.016 (0.046)
NFA/GDP	adj=0	−0.019 (0.004)	−0.012 (0.000)	−0.005 (0.037)
	adj=1	0.026 (0.063)	−0.0004 (0.956)	−0.003 (0.684)
NFA/GDP Inequality	adj=0	−0.008 (0.014)	−0.001 (0.801)	0.001 (0.475)
	adj=1	−0.008 (0.517)	−0.007 (0.361)	−0.006 (0.341)
Number of observations		6629	6629	6629
pseudo R^2		0.5308	0.5218	0.4280

Notes: Huber-White robust p-values in parenthesis, computed by the delta method.

horizontal label refers to the income level of the country represented in the y-axis, whereas the red vertical label refers to the income level of the country represented in the x-axis (a country-pair observation is represented by the point where a blue horizontal and a red vertical labels meet).[4] As this figure clearly illustrates, income heterogeneity is detrimental for union formation: the country pairs engaged in unions are those closer to the 45 degree line, ranging from poor country pairs such as those in the Economic and Monetary Community of Central Africa and those in the West African Economic and Monetary Union, to middle-income country pairs such as those in Mercosur,

[4]The countries corresponding to the labels are in Appendix A, together with the list of union arrangements. For any given pair, the specific country appearing on each axis was decided by the alphabetical order of the labels.

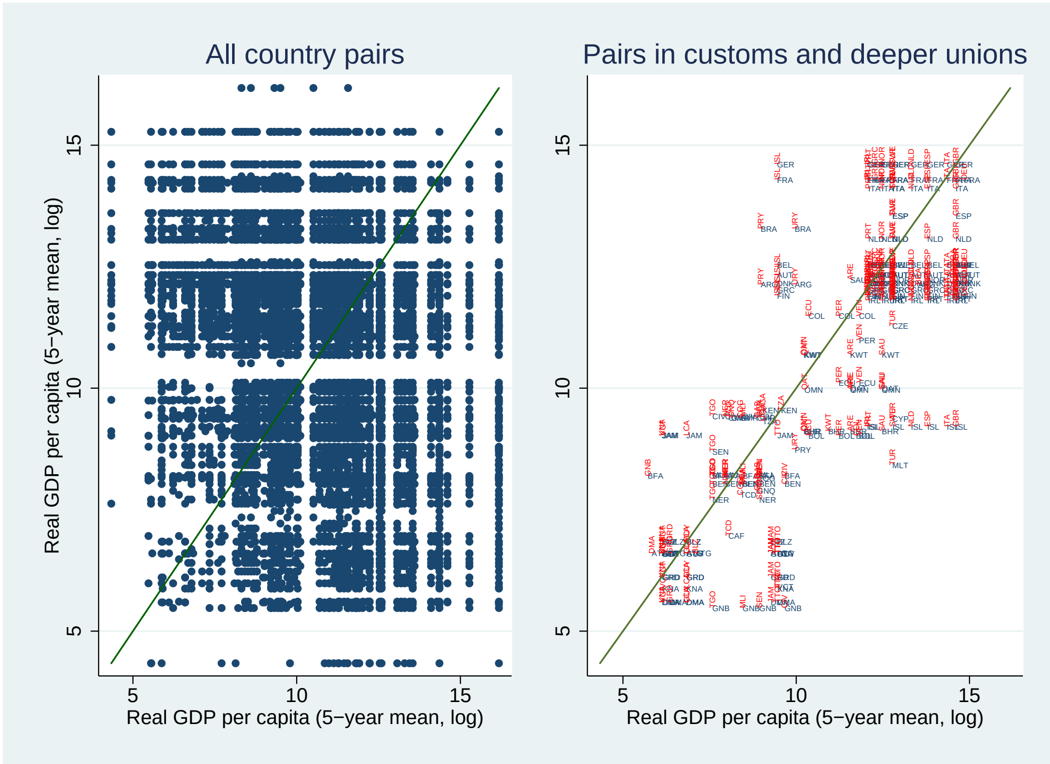

Figure 1: Incomes and union formation

to rich country pairs such as those in the European Union. This figure also shows clearly that higher income levels help union formation: there's a higher density of union-forming country pairs towards high income levels, mostly driven by the European Union countries.

The left panel of Figure 2 concentrates on net foreign assets over GDP.[5] It is again clear that heterogeneity in net foreign assets over GDP is detrimental for union formation. Consistently with the regression results, union-forming country pairs are not clearly those with higher levels of net foreign assets over GDP. There is a large concentration of unions towards the middle of the distribution.

There are two shortcomings of our empirical analysis which are worth pointing out. First, we treat newly-formed and continuing unions in 2004 both as instances of union formation, in line with Baier and Bergstrand (2009). This is obviously a caveat since, in reality, there is a likely bias towards the status-quo. That is, everything else constant, existing unions are more likely to continue than new unions to form. Unfortunately, the extremely small number of newly-formed

[5]To better visualize the data the left panel excludes Liberia, who had a level of net foreign assets over GDP of about -10.

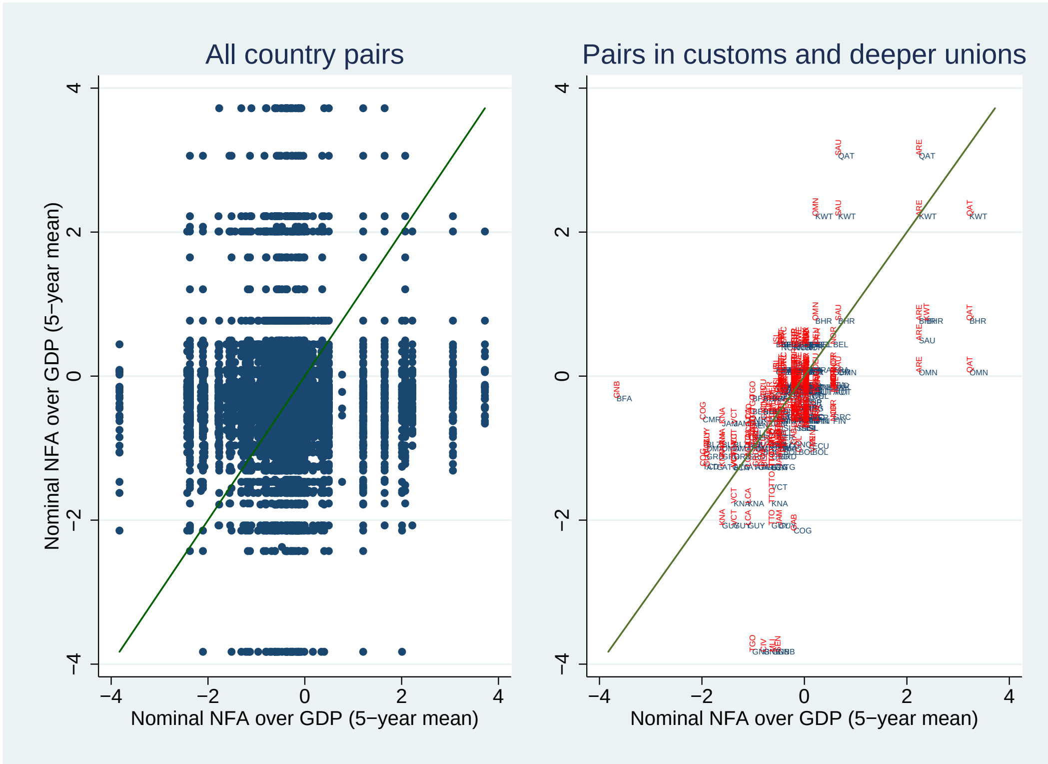

Figure 2: Net foreign assets over income and union formation

unions in any given year prevents us from concentrating only on new unions. Second, and in line with our theoretical model, we presume that union formation boils down to a bilateral decision. In reality, multi-country unions might not necessarily work in this fashion. When a multi-country union is being formed from scratch, countries presumably think about the average gain, and would not necessarily block union formation if they experience bilateral losses. However, in the case of accession into an existing union, it is conceivable that incumbent countries might be interested in vetoing the new member's entry if they experience a bilateral loss.

3 World economy

3.1 Model

Consider a world economy composed of a continuum of small open economies of measure one. Countries are identical ex-ante, and differ ex-post due to idiosyncratic endowment risk. Each period, a country receives an endowment of a non-storable consumption good. The endowment evolves over time according to a Markov chain with a finite number of states in the set Y. We denote by $y^t = \{y_s, y_{s+1}, \ldots, y_t\}$ the sequence of events from the initial time period $s < 0$ up to and including period t, and by $\pi(y^t)$ the probability of such sequence. The initial event $y^s = y_s$ is given and $\pi(y^s) = 1$. We denote by $\pi\left(y^t | y^\tau\right)$ the probability of y^t conditional on y^τ where $\tau \leq t$, and by $y^\tau \leq y^t$ the sequence y^τ which is a sub-root of y^t. We assume a law of large numbers holds in the cross-section of countries, which means there is no aggregate uncertainty.

Each country is populated by an infinitely-lived representative agent with preferences:

$$\sum_{t=s}^{\infty} \sum_{y^t \in Y^{t+1}} \beta^t \pi(y^t) u(c(y^t)), \tag{3.1}$$

where $\beta \in (0,1)$ is the subjective discount factor. The instantaneous utility u is increasing, strictly concave, and satisfies the Inada conditions: $\lim_{c \to 0} u'(c) = +\infty$ and $\lim_{c \to +\infty} u'(c) = 0$.

Countries cannot completely pool their income risk on world financial markets for two reasons. First, markets are incomplete: the menu of assets is exogenously restricted to a non-contingent one-period bond. A country's resource constraint is

$$c(y^t) + b(y^t) = y_t + (1+r) b(y^{t-1}), \tag{3.2}$$

where $b(y^t)$ is the demand for foreign bonds and r is the (time-invariant) world interest rate.

The second friction is that international lending contracts are imperfectly enforceable. At any time, a country is free to repudiate its foreign debt, the penalty being the permanent exclusion from any future trade. A country that contemplates debt repudiation faces a trade-off between current and future utility: defaulting implies higher current consumption, at a cost of lower future utility due to living in autarky. International lending contracts are self-enforcing, in the sense that borrowing countries always find the cost of repudiation larger than the benefit, and they always choose to repay. That is, allocations satisfy the following participation constraint:

$$\sum_{\tau=t}^{\infty} \sum_{y^\tau \in Y^{\tau+1}} \beta^{\tau-t} \pi\left(y^\tau | y^t\right) u(c(y^\tau)) \geq V_{aut}(y^t), \tag{3.3}$$

where $V_{aut}(y^t)$ is the value of entering financial autarky after the history y^t. It is the lifetime utility derived from consuming one's endowment each period from the history node y^t onwards:

$$V_{aut}(y^t) = \sum_{\tau=t}^{\infty} \sum_{y^\tau \in Y^{\tau+1}} \beta^{\tau-t} \pi\left(y^\tau | y^t\right) u\left((1-\phi) y_\tau\right).$$

The parameter $\phi \in [0,1]$ is a direct output cost associated with default. Such additional default penalty has been considered in the literature, and is typically motivated as a way to capture production disruptions that occur because of lack of access to international markets. As in Arellano (2008), our motivation is mainly quantitative. Without such penalty, the extent of borrowing and lending in the quantitative model is much lower than in the data.

The representative agent chooses contingent plans for consumption and foreign assets to maximize lifetime utility (3.1) subject to the resource constraint (3.2), the enforcement constraint (3.3), and a no-Ponzi game condition:

$$b(y^t) \geq -D, \tag{3.4}$$

where D is large enough that the constraint never binds in equilibrium.[6]

3.2 Recursive competitive equilibrium

We solve for the stationary recursive competitive equilibrium with solvency constraints. The state of the economy is characterized by net foreign bond holdings b and by the current endowment y. The problem of each country admits the following recursive formulation (see Bai and Zhang (2010) for a formal proof):

$$V(b, y) = \max_{c, b'} \left\{ u(c) + \beta \sum_{y'} \pi(y'|y) V(b', y') \right\} \tag{P0}$$

subject to:

$$\begin{aligned} c + b' &= y' + (1+r)b' \\ b' &\geq b^W(y). \end{aligned}$$

The state-contingent borrowing constraint b^W is the debt level such that for every possible state next period, the country is weakly better-off by repaying:

$$b^W(y) = \max_{y':\pi(y'|y)>0} \left\{ b_{y'} : V\left(b_{y'}, y'\right) = V_{aut}(y') \right\}. \tag{3.5}$$

This constraint allows countries to borrow as much as possible while preventing them from defaulting in any possible state next period. The state-contingency arises only when there exist future states that cannot be reached from current state. We assume $\pi(y'|y) > 0$ for all y, y', so that $b^W(y) = b^W$ for all $y \in Y$.

The autarky value V_{aut} is the solution to the following functional equation:

$$V_{aut}(y) = u\left((1-\phi)\, y\right) + \beta \sum_{y' \in Y} \pi(y'|y) V_{aut}(y'). \tag{3.6}$$

[6] The enforcement constraint does not prevent countries from running Ponzi schemes: an agent running a Ponzi game would never default on its debt, since this would prevent him from continuing running the scheme.

Let B be the set of net foreign bond levels, $S = B \times Y$ the state-space, and $\mathcal{A}_S$ the σ-Borel algebra of elements of S. We are now ready to define the stationary recursive competitive equilibrium of the world economy.

Definition. A *stationary recursive competitive equilibrium* is given by decision rules $c(b, y)$, $b'(b, y)$, a value function $V(b, y)$, a borrowing limit b^W, an interest rate r and a distribution Ψ of countries over individual states such that:

1. Given the world interest rate r and the borrowing limit b^W, the decision rules solve the recursive problem (P0) and V is the associated value function.

2. The borrowing limit b^W is not too tight, in the sense of satisfying equation (3.5) for all y.

3. The world credit market clears:
$$\int_S b'(b, y) d\Psi = 0.$$

4. The decision rules and the transition matrix of the endowment process induce a probability distribution P over the state space, $P: \ S \times \mathcal{A}_S \longrightarrow [0, 1]$, where:
$$P\left((b, y); A\right) = \sum_{y':(b'(b,y),y') \in A} \pi(y'|y)$$
is the probability of transiting from state (b, y) to a state in the set A.

5. The distribution Ψ is stationary and consistent with P:
$$\Psi(A) = \int_S P((b, y); A) d\Psi, \text{ for all } A \in \mathcal{A}_S.$$

3.3 Parameters and computation

Preferences are isoelastic:
$$u(c) = \frac{c^{1-\sigma}}{1-\sigma} \tag{3.7}$$
with a coefficient of relative risk aversion $\sigma = 1.5$. The subjective discount factor is selected so that the equilibrium world interest rate is 1%, yielding $\beta = 0.9773$.

The direct output penalty ensures that the cross-sectional standard deviation of the net foreign assets to GDP ratio equals 0.42, the average cross-sectional standard deviation obtained from the Lane and Milesi-Ferretti (2007) data set - we focus on a balanced panel of 110 countries over the 1970-2004 period. This yields $\phi = 0.0027$, or about a 0.3 percent yearly drop in output during default.

The endowment process is obtained from estimating a first-order autoregressive process:

$$\ln \tilde{y}_{it+1} = \rho \ln \tilde{y}_{it} + \sigma_\varepsilon \varepsilon_{it+1},$$

where $\ln \tilde{y}_{it} \equiv \ln y_{it} - \gamma_{0i} - \gamma_1 t$ and ε_{it+1} follows an i.i.d. $N(0,1)$. We estimate this process by pooling data on real output per capita from version 6.3 of the Penn World Tables (Heston, Summers, and Aten, 2009). We focus on a balanced panel of 111 countries over the 1960-2007 period. The point estimates of the key parameters are $\hat{\rho} = 0.977$ and $\hat{\sigma}_\varepsilon = 0.0615$. In the model we ignore the common trend and the country-specific means, normalizing every country's mean endowment to 1. We consider the common process

$$\ln y' = 0.977 \ln y + 0.0615\varepsilon',$$

with $\varepsilon' \sim$ i.i.d. $N(0,1)$. This process is discretized into a 5-state Markov chain using Rouwenhorst's (1995) procedure. The vector of endowment levels Y and the transition matrix $\Pi = [\pi_{yy'}]$ are reported in Table 2.

Y				
y_l	y_{lm}	y_m	y_{mh}	y_h
0.56	0.75	1.000	1.33	1.78

Π				
0.955	0.044	8×10^{-4}	6×10^{-6}	2×10^{-8}
0.011	0.955	0.033	4×10^{-4}	2×10^{-6}
10^{-4}	0.022	0.955	0.022	10^{-4}
2×10^{-6}	4×10^{-4}	0.033	0.955	0.011
2×10^{-8}	6×10^{-6}	8×10^{-6}	0.044	0.955

Table 2: Markov chain parameters

We briefly describe our numerical algorithm, the full details are provided in Appendix C.1. The outer loop solves for the interest rate that clears the world bond market. For given interest rate, we solve for debt limits which are not too tight, using the natural borrowing limit as the initial guess. Finally, for given interest rate and debt limits, we solve for the decision rules that solve the system of first-order conditions for the country's problem.

4 Economic union

We now describe the process of union formation in the model. We assume the world economy is in steady-state. At time $t = 0$, and without anticipating it, a pair of countries sitting in the world economy is offered the possibility of forming a union. We pick these two countries from the ergodic state-space of the world economy's stationary equilibrium. Each country is characterized by an initial state (b_{i0}, y_{i0}), $i = 1, 2$. We also assume that union formation is a once-and-for-all event, i.e. once a union is formed it cannot be dissolved in the future.

Within the union, we assume full enforcement, and complete financial markets.[7] Since a union is comprised of a finite number of countries (in this case two), there is still some endowment risk that the union would like to diversify away with the rest of the world. We assume union members still have access to world financial markets under the same conditions as before, i.e. by trading on non-contingent bonds subject to enforcement constraints. The union is like a small country in the world economy.

We assume the existence of a central authority in the union that coordinates the international trade and default decisions. Since union members coordinate their default decisions, there is a single union-wide enforcement constraint that applies to both countries at the same time. If the union defaults, all its members are permanently excluded from world markets, but they may still share endowment risk among them.

The union's endowment is determined by the realization of two independent and identically distributed endowment processes, one for each country. We denote it compactly by a two-dimensional vector $\bar{y}_t = (y_{1t}, y_{2t}) \in Y \times Y$, where the element $y_{it} \in Y$ is country i's endowment realization, $i = 1, 2$. With a slight abuse of notation, we also denote by π the transition probabilities for $\bar{y}$:

$$\pi(\bar{y}'|\bar{y}) = \prod_{i=1}^{2} \pi\left(y_i'|y_i\right),$$

where the $\pi\left(y_i'|y_i\right)$'s are displayed in Table 2.

[7] Completing markets may be achieved in a variety of ways, not just by increasing financial market sophistication. First, fiscal transfers in highly-integrated unions can achieve the same goal. Second, goods market liberalization may also complete markets. Cole and Obstfeld (1991) have shown that changes in terms of trade can go a long way towards insuring against idiosyncratic income risk; in some extreme cases trade in goods even provides all the necessary insurance, without the need for financial markets. Our view is that our abstract model captures in particular the risk-sharing benefits of goods market integration, even if it's not explicitly a model about commodity trade. Hence, our model's implications are more broadly relevant, even for actual unions whose explicit goal is not enhancing risk sharing.

4.1 Planner's problem

The allocation within the union is constrained-efficient, and can be obtained by solving a benevolent planner's problem. Although countries join the union with potentially different net foreign bond levels, only the aggregate net asset position matters for the planner's problem. Let $\bar{b}_0 = \sum_i b_{i0}$ and let λ_i be the weight the planner attaches to country i. The planner's problem is to solve for $\{c_i(\bar{y}^t)\}_{i=1,2}$ and $\bar{b}(\bar{y}^t)$, for all $\bar{y}^t$, $t \geq 0$, which maximize the weighted sum of the union partners' lifetime expected utilities

$$\sum_{i=1}^{2} \lambda_i \sum_{t=0}^{\infty} \sum_{\bar{y}^t} \beta^t \pi(\bar{y}^t) u(c_i(\bar{y}^t))$$

subject to the union-wide resource constraint

$$\sum_i c_i(\bar{y}^t) + \bar{b}(\bar{y}^t) = \sum_i y_{it} + (1+r)\bar{b}(\bar{y}^{t-1}),$$

for all $\bar{y}^t$, $t \geq 0$, to the union-wide enforcement constraint

$$\sum_i \lambda_i \sum_{\tau=t}^{\infty} \sum_{\bar{y}^\tau} \beta^{\tau-t} \pi(\bar{y}^\tau | \bar{y}^t) u(c_i(\bar{y}^\tau)) \geq W_{aut}^U\left(\bar{y}^t\right),$$

for all $\bar{y}^t$, $t \geq 0$, where

$$W_{aut}^U(\bar{y}^t) = \max_{\{c_i(\bar{y}^\tau)\}_i} \sum_i \lambda_i \sum_{\tau=t}^{\infty} \sum_{\bar{y}^\tau | \bar{y}^t} \beta^{\tau-t} \pi(\bar{y}^\tau | \bar{y}^t) u\left(c_i(\bar{y}^\tau)\right)$$

subject to

$$\sum_i c_i(\bar{y}^\tau) = (1-\phi) \sum_i y_{i\tau}, \text{ for all } \bar{y}^\tau, \tau \geq t,$$

for all $\bar{y}^t$, $t \geq 0$, and subject also to a no-Ponzi game condition

$$\bar{b}(\bar{y}^t) \geq -D, \tag{4.1}$$

for all $\bar{y}^t$, $t \geq 0$.

Apart from distributional issues, the planner's problem is similar to the problem of a country standing alone in the world economy, the main difference being that, because the partners' endowment processes are uncorrelated, the union faces an endowment process which is less volatile in the aggregate. Since markets are complete and contracts enforceable among union members, the lower aggregate endowment volatility translates into lower individual consumption volatility.

4.1.1 Reformulating the planner's problem

Under isoelastic preferences, the union planner's problem admits a simpler formulation which is very convenient. By Proposition 5 of Jeske (2006), aggregate borrowing and lending is independent of distributional issues. It follows that the planner's problem may be decomposed into two steps. In the first step, the planner solves for the optimal borrowing and lending of the union assuming a single representative country facing the aggregate endowment. In the second step, the planner redistributes the optimal aggregate consumption plan obtained from the first step among the two union partners.

Formally, the step 1 problem for the planner is

$$\max_{c(\bar{y}^t), b(\bar{y}^t)} \sum_{t=0}^{\infty} \sum_{\bar{y}^t} \beta^t \pi(\bar{y}^t) u(c(\bar{y}^t)) \tag{P1}$$

subject to the aggregate resource constraint

$$c(\bar{y}^t) + \bar{b}(\bar{y}^t) = \sum_{i=1}^{2} y_{it} + (1+r)\bar{b}(\bar{y}^{t-1}), \tag{4.2}$$

for all $\bar{y}^t$, $t \geq 0$, to the enforcement constraint

$$\sum_{\tau=t}^{\infty} \sum_{\bar{y}^\tau} \beta^{\tau-t} \pi(\bar{y}^\tau | \bar{y}^t) u(c(\bar{y}^\tau)) \geq V^U_{aut}(\bar{y}^t) \tag{4.3}$$

for all $\bar{y}^t$, $t \geq 0$, where

$$V^U_{aut}(\bar{y}^t) = \sum_{\tau=t}^{\infty} \sum_{\bar{y}^\tau | \bar{y}^t} \beta^{\tau-t} \pi(\bar{y}^\tau | \bar{y}^t) u\left((1-\phi)\sum_i y_{i\tau}\right),$$

for all $\bar{y}^t$, $t \geq 0$, and to the no-Ponzi game condition (4.1).

Given the optimal plan $c(\bar{y}^t)$ from step 1, step 2 solves for the optimal distribution of aggregate consumption among the union partners. Formally, the step 2 problem is

$$\max_{\{c_i(\bar{y}^t)\}} \sum_i \lambda_i \sum_{t=0}^{\infty} \sum_{\bar{y}^t} \beta^t \pi(\bar{y}^t) u(c_i(\bar{y}^t)) \tag{P2}$$

subject to

$$\sum_i c_i(\bar{y}^t) = c(\bar{y}^t),$$

for all $\bar{y}^t$, $t \geq 0$.

With isoelastic preferences, the step 2 problem admits a simple, explicit solution. It is relatively easy to show that

$$c_i(\bar{y}^t) = \alpha_i c(\bar{y}^t) \tag{4.4}$$

where $\alpha_i \equiv \lambda_i^{1/\sigma} / \sum_j \lambda_j^{1/\sigma}$, for $i = 1, 2$. That is, individual consumption is a constant fraction of aggregate consumption. The fraction is increasing in the country's welfare weight.

Similarly to Section 3.2, the step 1 planner's problem admits a recursive formulation:

$$V^U(\bar{b}, \bar{y}) = \max_{c, \bar{b}'} \left\{ u(c) + \beta \sum_{\bar{y}'} \bar{\pi}(\bar{y}'|\bar{y}) V^U(\bar{b}', \bar{y}') \right\} \tag{P1$'$}$$

subject to

$$\begin{aligned} c + \bar{b}' &= \sum_i y_i' + (1+r)\bar{b} \\ \bar{b}' &\geq \bar{b}^U(\bar{y}) \end{aligned}$$

where

$$\bar{b}^U(\bar{y}) = \max_{\bar{y}' : \bar{\pi}(\bar{y}'|\bar{y}) > 0} \left\{ b_{\bar{y}'} : V^U\left(b_{\bar{y}'}, \bar{y}'\right) = V^U_{aut}(\bar{y}') \right\} \tag{4.5}$$

and where $V^U_{aut}(\bar{y})$ solves

$$V^U_{aut}(\bar{y}) = u\left((1-\phi)\sum_i y_i\right) + \beta \sum_{\bar{y}'} \pi(\bar{y}'|\bar{y}) V^U_{aut}(\bar{y}').$$

Given (4.4), the value for country i of belonging to a union with country j is

$$V_i^U(\bar{b}, \bar{y}) = \alpha_i^{1-\sigma} V^U(\bar{b}, \bar{y}). \tag{4.6}$$

4.2 Competitive equilibrium

To perform our welfare analysis, we still need to recover the planner's welfare weights as a function of the initial pair of union partner states.

We use Negishi's (1960) iterative method to compute these welfare weights. This well-known method exploits the first welfare theorem, which allows us to obtain the competitive equilibrium allocation as the solution to the planner's problem for a given set of welfare weights. By requiring that the planner's allocation be affordable under the equilibrium prices, we obtain the unique pair of welfare weights that lead to the competitive equilibrium allocation associated with a given set of initial states.

We need to consider a decentralization of the constrained efficient allocation. We consider a competitive equilibrium with tax subsidies, in line with Kehoe and Perri (2004) and Wright (2006). The decentralization works as follows. Within the union, countries trade a complete set of Arrow securities. In world credit markets, they trade freely on non-contingent bonds. However, a central government authority in the union taxes each country's income in a lump-sum fashion, and uses the proceeds to subsidize asset purchases. The government's tax and transfer policy is designed to support the constrained-efficient allocation. A subsidy is required to encourage union partners to save in those states when they would be inclined to default. Our procedure is described in more detail in Appendix B.

4.3 Discussion

Several features of union formation in our model are worth discussing. The role of initial conditions when computing the welfare gains from financial market integration is a crucial feature of our analysis. Whether a country is rich or poor at the time of union formation is a key determinant of the sign of the welfare gains. In the international risk-sharing literature, the role of initial conditions has sometimes been sidestepped (Cole and Obstfeld, 1991; van Wincoop, 1999; Athanasoulis and van Wincoop, 2000, either impose symmetry, or look at a representative country), whereas in other papers (van Wincoop, 1994; Lewis, 2000; Imbs and Mauro, 2008) it is allowed to play a role. Differently from this literature, however, in our model union formation may entail a welfare loss. This generates the potential for disagreement about union formation. We exploit this by requiring that unions be formed only when both partners experience a welfare gain, given the initial conditions set in the world economy. That is, union formation in our model requires unanimity.

For a large set of country pairs in our model, unions only lead to potential Pareto improvements, with one country losing. This raises the possibility of introducing *side payments* to compensate the losers. Our analysis abstracts from such transfer schemes. In our setup, wealth would need to be redistributed away from poor and toward rich partners. We suspect the implementation of such schemes would face strong opposition in poor countries. Moreover, we do not have evidence from actual integration arrangements suggesting such schemes have taken place.[8] Finally, we believe it is more appropriate to focus our analysis strictly on the benefits from risk-sharing, separately from side-payments.

[8] In the European Union, the Cohesion Fund is a transfer scheme that takes the exact opposite form: resources are transferred from rich to poor members.

Rather than implementing a pure transfer scheme, the two partners could instead agree ex-ante on distorting the baseline union allocation, tilting it to the benefit of rich partners. Formally, one would impose *ex-ante participation constraints*, at the time of union formation, such that every partner may potentially benefit from it. This would increase the likelihood of union formation among heterogeneous partners, at the expense of future risk-sharing benefits. Presumably, such arrangement would be easier to implement, on political economy grounds, compared to a pure transfer scheme. We think it would be very interesting and relevant to extend our analysis along this dimension.

We considered unions with *centralized international trade and default decisions*. An alternative setting is one in which each individual member country unilaterally decides whether to default. Jeske (2006) provides an analysis of this situation. As Section 4.1 makes clear, a major advantage of our centralized setting is analytical convenience, since it does not require solving directly for the market allocation. Note however that with decentralized default, potentially defaulting union members presume continued indirect access to world markets, by using the remaining non-defaulting members as intermediaries. This increases the incentives to default, and therefore tightens borrowing limits within the union relative to centralized default. All else constant, union formation is thus even less likely under decentralized compared to centralized default. Our analysis can be thought of as giving the best chance for union formation.

For tractability, our analysis restricts attention to *two-country unions*. In our model, since endowment risk is purely idiosyncratic, additional partners would be potentially beneficial to the union since they would further enhance risk-sharing opportunities. However, solving the frictions among union members is also likely to become more difficult and costly as the number of partners increases. This is precisely the starting premise of our paper, that solving frictions is easier at a smaller scale. Our model could be extended by introducing a cost of union formation that is increasing with the number of countries.[9] Such a setting would deliver implications for both the number and the type of countries most likely to form a union. We leave the analysis of these interesting implications to future research.

Finally, a country pair contemplating union formation is given a take-it-or-leave choice at time

[9] Imbs and Mauro (2008) find that, regarding the benefit side alone, most risk-sharing gains would be achievable in unions of seven member-countries or less. Further, in our model it is difficult for a large number of countries to all agree about union formation. This suggests that even very small costs would be sufficient to generate to small-scale arrangements.

0. If the union is formed, it is assumed to be forever enforced. Our analysis abstracts away from the important issue of sustainability of the economic union. Although *union breakups* are very rare in the data, they can be ex-post optimal in our model, depending on the endowment realization. Without an enforcement technology, sustaining the union would require distorting the optimal allocation, to ensure that the relevant ex-post participation constraints are met. In some cases this might not be possible, leading to a breakup of the union. See Fuchs and Lippi (2006) for an analysis of the sustainability of monetary unions with some of these features in an incomplete contract setting.

5 Results

Our goal is to characterize which country pairs find it individually rational to form a union. The main benefit of union formation is the possibility of sharing risk with a partner. There are also costs, however. First, default becomes more attractive for union members, since they may still share risk upon default. As a result $b_i^W < \bar{b}_i^U$, i.e. borrowing constraints become tighter in the union.[10] In our benchmark calibration, the borrowing limit increases from $b_i^W = -0.2$ in the world economy, to $\bar{b}_i^U = \bar{b}^U/2 = -0.191$ in the union, on a per country basis.

Second, in asymmetric unions, poorer country members tend to borrow heavily from the rest of the world, and exhaust the whole union's borrowing limit. This imposes a cost on richer countries, which find themselves more frequently borrowing-constrained compared to standing alone in the world economy. Although being part of an asymmetric union tends to be beneficial for poorer members, it also tends to generate losses for richer countries. Our model will therefore produce a bias against forming asymmetric unions.

We now turn to a more detailed analysis of union formation. We compute the welfare gain for each country of forming a union with a specific partner in terms of consumption equivalents. That is, as the percentage increase in consumption, constant across time and states of nature, that leaves the country indifferent between standing alone in the world economy and forming the union.

Consider two countries sitting in the world economy at time 0, with states (b_{i0}, y_{i0}), $i = 1, 2$. If they form a union, the initial aggregate state is $(\bar{b}_0, \bar{y}_0)$, with $\bar{b}_0 = b_{10} + b_{20}$ and $\bar{y}_0 = (y_{10}, y_{20})$. Let $c^W(b_{i0}, y_{i0})$ represent a state-contingent consumption stream for country i in the world economy,

[10]Theoretically, $b_i^W \geq \bar{b}_i^U$ may also obtain. This would be the case if the value of union formation was high enough relative to the value of staying alone in the world economy, compared to the difference in the outside options. However, this was never the case in our quantitative analysis.

from state (b_{i0}, y_{i0}) onwards. Let $c_i^U\left(\bar{b}_0, \bar{y}_0\right)$ represent a state-contingent consumption stream for country i if both i and j decide to form a union at time 0. Let $U(c^W(b_{i0}, y_{i0}))$ and $U\left(c_i^U\left(\bar{b}_0, \bar{y}_0\right)\right)$ denote the expected lifetime utility derived from these consumption streams. Now denote by $(1+\mu_{ij})c^W(b_{i0}, y_{i0})$ the consumption stream derived from $c_i^W(b_{i0}, y_{i0})$, where every state-contingent consumption level is increased by μ_{ij} percent. The welfare gain for country i of forming a union with country j is the μ_{ij} that solves:

$$U\left((1+\mu_{ij})c^W(b_{i0}, y_{i0})\right) = U\left(c_i^U\left(\bar{b}_0, \bar{y}_0\right)\right),$$

or, with isoelastic preferences as in (3.7),

$$\begin{aligned} \mu_{ij} &= \left[\frac{U\left(c_i^U\left(\bar{b}_0, \bar{y}_0\right)\right)}{U\left(c^W(b_{i0}, y_{i0})\right)}\right]^{\frac{1}{1-\sigma}} - 1 \\ &= \left[\frac{V_i^U\left(\bar{b}_0, \bar{y}_0\right)}{V(b_{i0}, y_{i0})}\right]^{\frac{1}{1-\sigma}} - 1, \end{aligned} \tag{5.1}$$

where the value functions have been defined in (P0) and (4.6). Notice that our welfare numbers incorporate transitional dynamics.

We next study the separate roles of wealth heterogeneity and wealth levels for union formation.

5.1 Role of wealth heterogeneity

Figure 3 displays the welfare gain for country 1 of forming a union with country 2, as a function of country 1 and country 2's initial net foreign asset levels. The figure is conditional on country 1 being endowment-rich (starting the union formation process with endowment y_{mh}) and country 2 being endowment-poor (endowment y_{lm}).

Several observations emerge. First, country 1 experiences a welfare loss for a large range of net foreign asset levels. The equilibrium welfare gains, restricted to the ergodic space, range from -5.5% to 22%, with a mean of 2.8%. These are sizable welfare gains from union formation. Comparing with the literature on the welfare gains from international risk-sharing, the average gain is toward the lower end of the estimated range, as summarized by van Wincoop (1999), but higher than in papers reporting gains of at most 0.5% such as Cole and Obstfeld (1991), Backus, Kehoe, and Kydland (1992), Obstfeld (1994a), Tesar (1995), and Mendoza (1995).

Second, Figure 3 shows that country 1's welfare gain is always increasing in the partner's net foreign assets. Third, country 1's welfare gain is increasing in own net foreign assets only if the partner's is sufficiently low;[11] otherwise, if the partner is rich, the welfare gain is monotonically decreasing in own net foreign assets. Put together, the last two observations suggest the key determinant for union formation is the amount of the resources the partner has: a country would like to belong to a rich country club, especially if it's a poor one.

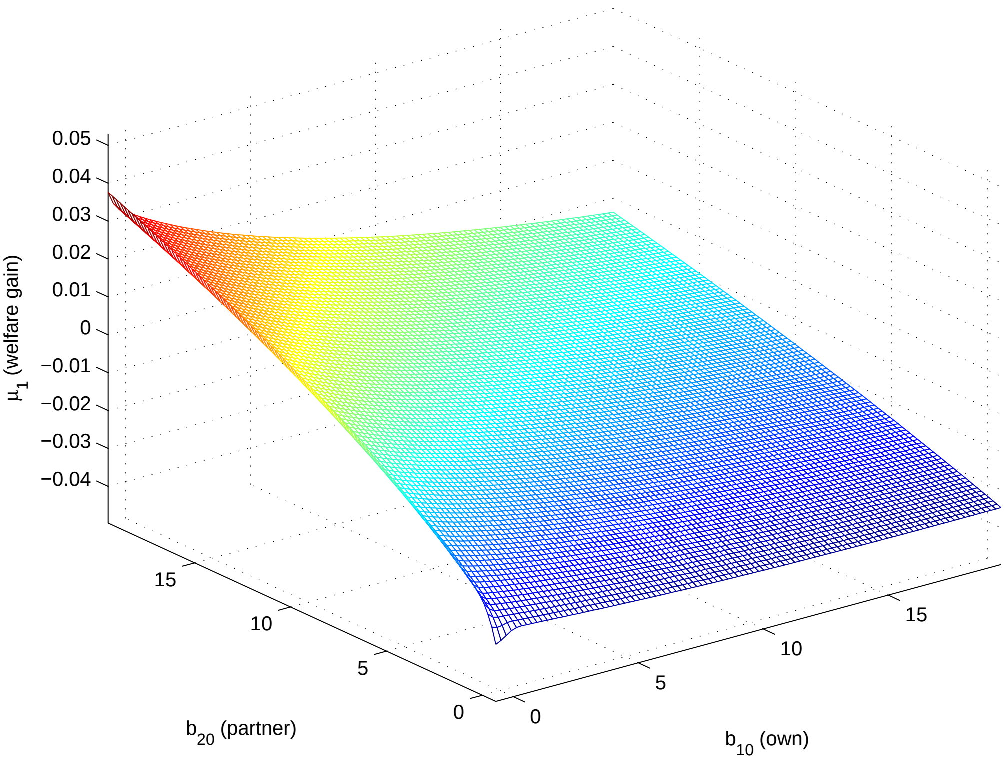

Figure 3: Welfare gain from union formation

Figure 4 displays the agreement areas, i.e. the set of initial country states for which both countries would experience a welfare gain, and thus agree to form a union. To streamline the exposition, Figure 4 is restricted to endowment levels in $\{y_{lm}, y_m, y_{mh}\}$. For states above the solid lines, country 1 would improve welfare by forming a union with country 2, and similarly for country 2 for states below the dashed lines. The agreement areas are therefore represented by the light-shaded areas.

Superimposed on Figure 4 is also an area representing the ergodic space for net foreign asset positions in the world economy, $b_{10}, b_{20} \in [-0.2, 4.72]$.[12] This is the dashed square located inside

[11] Although not apparent from the Figure 3, the welfare gain is actually non-monotonic in own net foreign assets if the partner's is low enough. The reason will become clear when we discuss Figure 5.

[12] Since the average endowment is equal to 1, these quantities correspond also to net foreign assets to average output ratios.

each figure. Notice the role played by the world steady-state equilibrium in our analysis of union formation. It determines both the world interest rate faced by the union, and also the relevant subset of country pairs that are faced with the option of union formation.

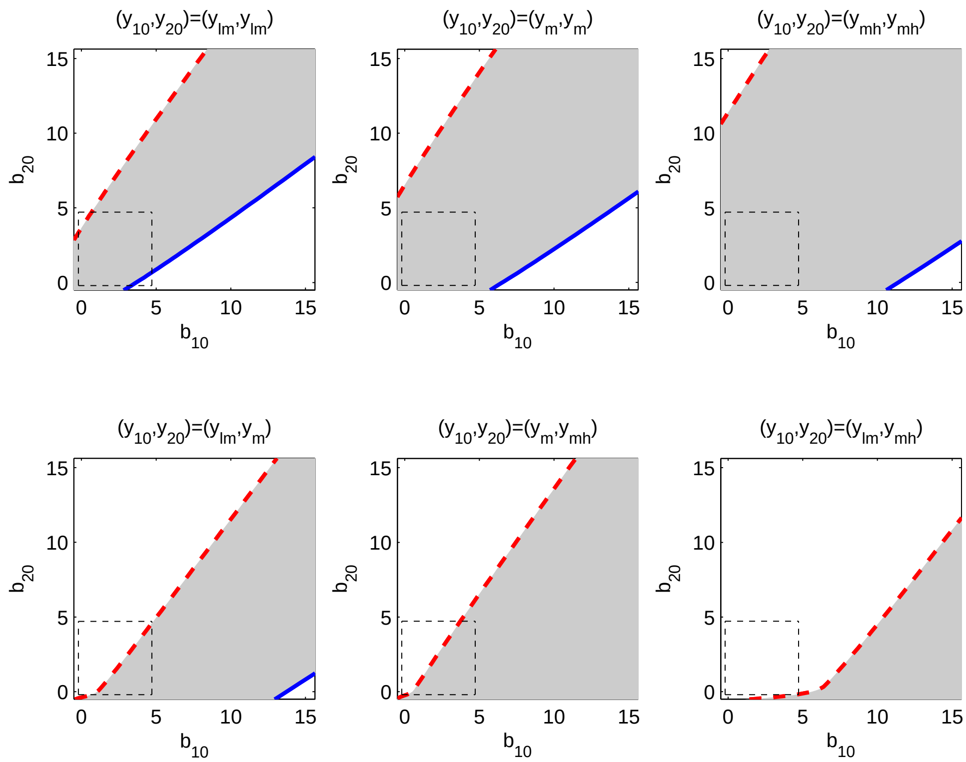

Figure 4: Agreement areas (country 1: solid, country 2: dashed)

We begin with the first row of Figure 4. Potential union members have identical initial endowments, but potentially different wealth levels. The figure shows, first, that unions tend to be formed between countries sufficiently homogeneous in terms of initial wealth. Along the 45 degree line, and restricted to the ergodic space, countries always reach an agreement. The disagreement area exists when wealth levels are sufficiently different from each other. Second, whenever partners disagree, the rich are the ones with a potential welfare loss. They are the ones preventing union formation.

Turning now to the bottom row of Figure 4, which corresponds to asymmetric initial endowments, we see that endowment heterogeneity makes it very difficult for countries to agree to form a union. In the extreme case when endowment levels are in $\{y_{lm}, y_{mh}\}$, an agreement is never reached. Although country 1, the endowment-poor country, would always benefit from union formation (the ergodic space is always above the solid line), this is not the case for country 2, the endowment-rich country. Only a sufficiently asset-poor country 2 would like to form a union with an endowment-poor country 1.

The bottom line is that country homogeneity, either in terms of net foreign assets or endowments, is a key determinant of union formation. Unions are more likely to form among similar countries. The key mechanism underlying partner disagreement is the effect the union generates on the probability of becoming constrained in the future.

To better understand this mechanism we turn to Figure 5. This figure displays the difference between the probability of becoming credit-constrained in a union and the probability of becoming credit-constrained while standing alone in the world economy, during the first 100 periods starting from today.[13] This is computed for each initial level of net foreign assets of the reference country (labeled "own" in the figure) and of any given potential union partner, conditional on the endowment being equal to y^h for country 1 (relatively rich) and y_m for country 2 (relatively poor).

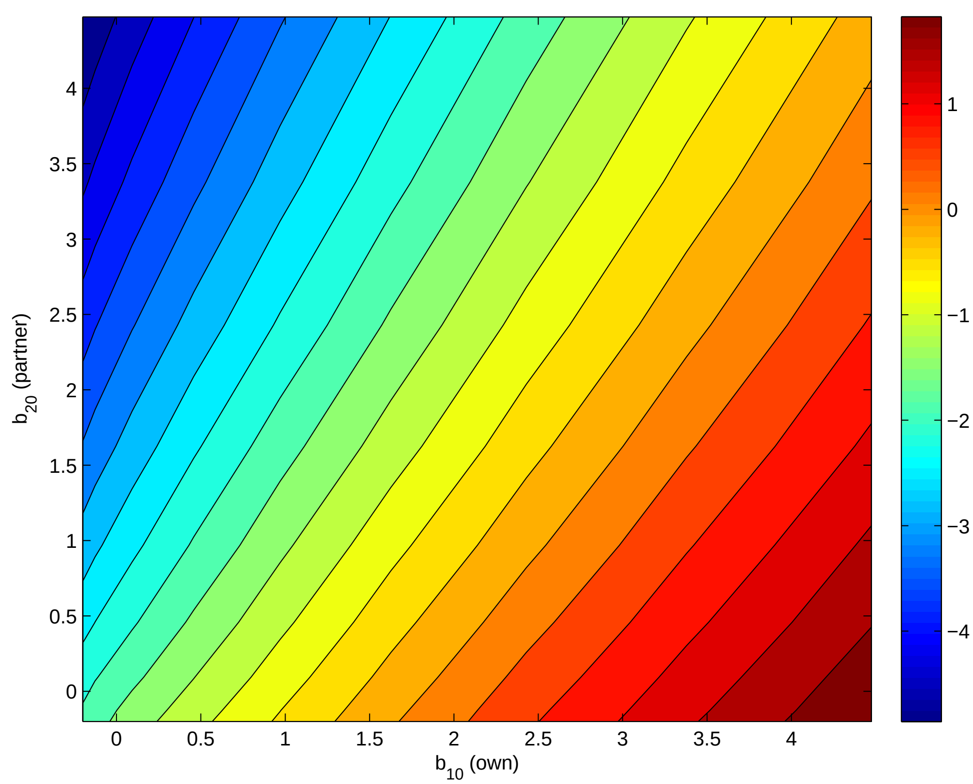

Figure 5: Excess probability of becoming credit constrained in the union

Several observations emerge. First, the excess probability is negative for a large set of states.

[13]Our focus on the short run stems from the fact that we wish to understand the welfare comparisons underlying Figure 4, and individuals obviously discount the future. The excess probability in Figure 5 is in percentage points.

This is in spite of tighter borrowing limits in the union: countries are better insured in the union, hence borrow less in world credit markets and hit the constraint less often compared to standing alone. Second, the excess probability becomes more negative when the reference country is poorer and the partner richer. Third, the excess probability becomes positive when the reference country is richer and the partner poorer. These are precisely the areas of disagreement we identified earlier, illustrating the importance of our mechanism: asymmetric unions benefit poor countries at the expense of rich, via changes in the likelihood of becoming credit-constrained following union formation.

5.2 Role of wealth levels

We now turn to the role of total wealth (net foreign assets plus endowment) levels. From the first row of Figure 4, we see that a larger union-wide endowment favors union formation. First because, as we move from the left to the right panel, the agreement area fills a larger area of the ergodic space. Second because the agreement areas get wider for larger wealth levels, which is particularly noticeable when conditional on (y_l, y_l).

Figure 5 once again helps us understand the basic mechanism. As we move along any line starting from the lower left corner of the figure, the excess probability that country 1 becomes credit-constrained in the union decreases. When both partners are richer they are farther away from their borrowing constraints, and are thus less likely to face the type of disagreement that we illustrated in the previous section.

We summarize the discussion of this and the previous subsection with the following. Unions are more likely to be formed:

1. the wealthier the partners, and
2. the more homogeneous the partners,

either in terms of initial endowment or net foreign assets.

5.3 Quantitative implications

To explore the quantitative implications of the model, we compute the probability of union formation conditional on different regions of the state-space.[14] We ask: What is the probability that two

[14] An alternative procedure would be to run a probit-gravity regression on artificial data which would be the exact analogue of the one in Section 2, except that the terms involving geography and scale would be excluded.

randomly-picked countries from particular subsets of the world distribution agree to form a union?

In selecting subsets of the ergodic space, we focus on the top and bottom terciles for output (respectively defined as $Y_h = [y_{2/3}, y_{\max}]$ and $Y_l = [y_{\min}, y_{1/3}]$) and net foreign-assets over GDP (respectively defined as $B_h = [(b/y)_{2/3}, (b/y)_{\max}]$ and $B_l = [(b/y)_{\min}, (b/y)_{1/3}]$). We define such sets in the exact same way both in the actual data and in the model. Since the results are similar across our empirical definitions of unions, in the actual data we restrict attention to customs unions or deeper arrangements.

We restrict attention to only three subsets, with the aim of capturing the key implications we drew from Figure 4. More specifically, take country pairs defined by their current output and net foreign assets over GDP.[15] We consider "Rich" country pairs (both in the set $Y_h \times B_h$), "Poor" country pairs (both in the set $Y_l \times B_l$), and "Unequal" country pairs (one in the set $Y_h \times B_h$ and the other in $Y_l \times B_l$). We also compute the "Unconditional" probability of union formation.

	Data	Data, no extreme differences	Model
Rich	71%	68%	67%
Poor	20%	30%	57%
Unequal	0%	0%	1%
Unconditional	32%	41%	29%

Table 3: Conditional Probabilities of Union Formation

Our results are summarized in Table 3. The first column pertains to the data. Since our model abstracts from geography, our "Data" is restricted to country pairs sharing a border. About 32% of all country pairs are part of a customs union or deeper arrangement in 2004. This number is 20% conditional on poor country pairs, and 71% conditional on rich country pairs. The data does not feature unions among unequal pairs.

For the purpose of comparing our results to the data, recall that our calibrated income process from Section 3.3 abstracts from the estimated country fixed effects. This means our model-generated income differences are lower than in the raw data. As it turns out, this is the case even only among common border countries. To address this shortcoming, our second column of Table 3 further

Unfortunately, due the nonlinear nature of the regression model, the marginal effects would be hard to compare.

[15] For the reasons explained in Section 2, by "current" levels we actually mean five-year averages.

restricts the data to those common border countries with pairwise income differences lower than the top 1/3. When we do this, the model-generated income differences are quantitatively very similar to those in the restricted data set. The conditional probabilities of union formation, however, are qualitatively similar to the first column. The main difference is that there are more unions being formed, especially among poor country pairs.

The third column contains de conditional probabilities of union formation in the model. The model is qualitatively consistent with the data in the sense that relatively few unions are formed, and the ones that do get formed are mostly among similar countries, and also rich ones. This confirms the analysis of Section 5. These probabilities resemble reasonably closely the ones in the data from a quantitative standpoint. The main discrepancy is that our model implies low wealth levels are not nearly as detrimental to union formation compared to the data.

We conclude that our model seems to provide a reasonably accurate description of the incentives for union formation, namely the role of wealth levels and wealth inequality.

6 The European Union: a short digression

There are two aspects of the European Union experience which appear to be consistent with our model. First, the successive accession waves happened after an important degree of income convergence has taken place between accessing and member countries. Our model says that this is an important condition for union formation. Figure 6 illustrates this fact.[16] The left panel represents the accession of Greece in 1981, and Portugal and Spain in 1986. Together with the real income of the accessing countries, the figure also plots the mean real income of the countries which were members by 1980. A very significant degree of convergence has occurred before these Southern European countries joined the European Union (European Economic Community by then), in fact to a much larger extent than the degree convergence that took place afterwards. The right panel of Figure 6 documents the accession of the Eastern European block in 2004. All these countries without exception have experienced a significant degree of convergence before joining the European Union.

It is also important to point out that both the Southern and the Eastern European countries were able to join the European Union when they did due to the *sine qua non* removal of political obstacles: the Southern European countries became democracies in 1974/5, and the Eastern block

[16]The countries corresponding to the labels in the legend are indicated in Appendix A.

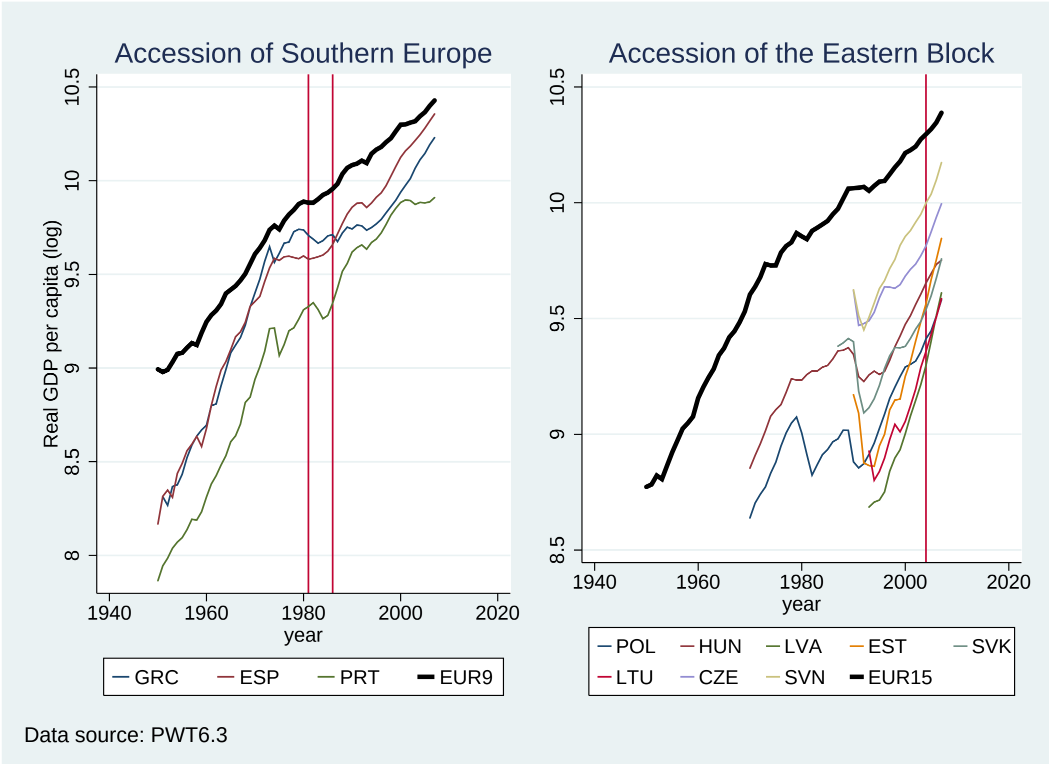

Figure 6: Income levels upon accession into the European Union

around 1990. While these political considerations were obviously central, economic considerations were central too. Accessing countries were required to implement major free-market economic reforms as a condition for membership. These reforms were no doubt important for the subsequent economic performance of accessing countries; we would say also for the success and the stability of the European Union.

The second aspect of the European Union experience which seems consistent with our model is related to the current crisis involving Greece and Ireland, and to a lesser extent Portugal and Spain. All these countries became highly indebted in foreign markets in recent times, in large measure benefiting from German credibility and low interest rates. At the same time, we do not see any indication that these countries contemplate abandoning the euro area. Instead, it is the rich countries, most notably Germany, who are unhappy about providing aid to the Southern European countries.[17] Our model predicts that poor countries borrow a lot once members of a union, and

[17]See for example the Dec 2, 2010 article in *The Economist* titled "Germany and the euro: We don't want no transfer union." A short quotation illustrates the main gist: "In adopting the euro the Germans thought they were joining a condominium, in which every member would keep order on their own property, and not a messy commune. Now the crisis threatens that understanding."

says that instability within the union would arise precisely in this form, with rich countries having a preference for breaking up.

7 Conclusion

We have developed a quantitative theory of economic integration based on the incentives to share income risk. We have modeled an economic union as a small-scale arrangement that solves the frictions that otherwise limit the extent of risk sharing in the world economy.

Our model emphasizes not only the risk-sharing benefits of union formation, but also its costs. One cost is that union members as a whole will not be able to borrow as much as in the world economy. This is because unions have larger incentives to default. Another cost is for rich countries in asymmetric unions. Poor countries tend to exhaust the union's credit limit, imposing a cost on rich countries. Our model implies that economic integration should not happen very often, and when unions do get formed it is mostly among rich and homogeneous countries. These features appear to be consistent with real-world arrangements.

Our paper has abstracted from several dimensions of country heterogeneity which could be potentially important for union formation based on risk-sharing. We have assumed countries are characterized by common and independent income processes. In reality, shocks tend to be correlated among subsets of countries, which would work against union formation in our model.[18] Further, there is large cross-country heterogeneity in income risk, with poorer countries being more volatile (e.g. Acemoglu and Zilibotti (1997)). In our model, this could potentially increase the likelihood of union formation among poor countries.[19] Finally, there are also differences in country size. All these issues deserve further scrutiny.

Our paper has also focused on just one particular dimension of economic integration, the sharing of risk. It would be interesting to consider other important dimensions of economic integration for small scale arrangements, namely liberalizing goods flows (Melitz (2003), Alvarez and Lucas (2007)), labor flows (Klein and Ventura (2007)), and investment flows (Castro (2005), Gourinchas and Jeanne (2008), Burstein and Monge-Naranjo (2009), McGrattan and Prescott (2009)).[20]

[18] Correlated shocks is instead traditionally emphasized as a motivation for the formation of currency unions.

[19] A similar implication follows from Imbs and Mauro's (2008) analysis.

[20] Further dimensions of small scale economic integration which received some attention in the recent literature include adopting a common currency (Alesina and Barro (2002)) and coordinating public policy (Alesina, Angeloni, and Etro (2005)).

A Data

A.1 Countries

The full sample of 136 countries that we use in the regression analysis of Section 2 includes: Algeria, Angola, Antigua and Barbuda, Argentina, Australia, Austria, Bahrain, Bangladesh, Belgium, Belize, Benin, Bhutan, Bolivia, Brazil, Bulgaria, Burkina Faso, Burundi, Cameroon, Canada, Central African Republic, Chad, Chile, China, Colombia, Comoros, Congo D.R., Congo Rep., Costa Rica, Côte d'Ivoire, Cyprus, Czech Republic, Denmark, Djibouti, Dominica, Dominican Republic, Ecuador, Egypt, El Salvador, Equatorial Guinea, Ethiopia, Fiji, Finland, France, Gabon, Gambia, Germany, Ghana, Greece, Grenada, Guatemala, Guinea, Guinea-Bissau, Guyana, Haiti, Honduras, Hong Kong, Hungary, Iceland, India, Indonesia, Iran, Ireland, Israel, Italy, Jamaica, Japan, Jordan, Kenya, Kiribati, Korea, Kuwait, Lao PDR, Liberia, Madagascar, Malawi, Malaysia, Mali, Malta, Mauritania, Mauritius, Mexico, Mongolia, Morocco, Mozambique, Nepal, Netherlands, New Zealand, Nicaragua, Niger, Nigeria, Norway, Oman, Pakistan, Panama, Papua New Guinea, Paraguay, Peru, Philippines, Poland, Portugal, Qatar, Romania, Rwanda, Samoa, Saudi Arabia, Senegal, Seychelles, Sierra Leone, Singapore, Solomon Islands, South Africa, Spain, Sri Lanka, St. Kitts and Nevis, St. Lucia, St. Vincent and the Grenadines, Sudan, Sweden, Switzerland, Syrian Arab Republic, Tanzania, Thailand, Togo, Trinidad and Tobago, Tunisia, Turkey, Uganda, United Arab Emirates, United Kingdom, United States, Uruguay, Venezuela, Vietnam, Yemen, Zambia, and Zimbabwe.

A.2 Regional Agreements

The list of regional trade agreements in force in 2004 that we use in the regression analysis of Section 2, by type, and their country composition, is (PWT codes in parenthesis, only for countries in our sample):

- **Economic Unions.**
  - *Economic and Monetary Community of Central Africa:* Cameroon (CMR), Central African Republic (CAF), Chad (TCD), Congo D.R. (ZAR), and Equatorial Guinea (GNQ).
  - *Euro zone (EU12):* Austria (AUT), Belgium (BEL), Luxembourg (LUX), Finland (FIN), France (FRA), Germany (GER), Greece (GRC), Ireland (IRL), Italy (ITA), Netherlands

(NLD), Portugal (PRT), and Spain (ESP).

  - *West African Economic and Monetary Union:* Benin (BEN), Burkina Faso (BFA), Guinea-Bissau (GNB), Cote d'Ivoire (CIV), Mali (MLI), Niger (NER), Senegal (SEN), and Togo (TGO).

- **Common Markets.** In addition to all Economic unions:
  - *East African Community:* Kenya (KEN), Tanzania (TZA), and Uganda (UGA).
  - *European Economic Area (EEA):* all the *EU12* countries listed above, plus the non-Euro zone countries of Denmark (DNK), Finland (FIN), Sweden (SWE), and the United Kingdom (GBR), plus the European Free Trade Area (*EFTA*) countries of Iceland (ISL), Liechtenstein, and Norway (NOR).

- **Customs Unions.** In addition to all Common Markets:
  - *Andean Community:* Bolivia (BOL), Colombia (COL), Ecuador (ECU), Peru (PER), and Venezuela (VEN).
  - *Caribbean Community:* Antigua and Barbuda (ATG), Bahamas, Barbados, Belize (BLZ), Dominica (DMA), Grenada (GRD), Guyana (GUY), Jamaica (JAM), Montserrat, Saint Kitts and Nevis (KNA), Saint Lucia (LCA), Saint Vincent and the Grenadines (VCT), Suriname, and Trinidad and Tobago (TTO).
  - *Eurasian Economic Community:* Belarus, Kazakhstan Kyrgyzstan, Russia (RUS), and Tajikistan.
  - *EU Customs Union:* Turkey (TUR), Andorra, San Marino, and Monaco, plus all the *EEA* countries listed above, except the 3 which are only part of *EFTA*.
  - *Gulf Cooperation Council:* Bahrain (BHR), Kuwait (KWT), Oman (OMN), Qatar (QAT), Saudi Arabia (SAU), and United Arab Emirates (ARE).
  - *Southern Common Market (Mercosur):* Argentina (ARG), Brazil (BRA), Paraguay (PRY), and Uruguay (URY).
  - *South African Customs Union:* Botswana, Lesotho, Namibia, South Africa (ZAF), and Swaziland.

B Decentralization

We decentralize the planner's allocation as a competitive equilibrium with tax subsidies on saving. Our decentralization scheme is an adaptation of Kehoe and Perri (2004) and Wright (2006).[21] Within the union, countries trade a complete set of Arrow securities. In the world market, they trade freely on a riskless one-period bond. A central government authority in the union implements a tax and transfer scheme, designed to support the constrained-efficient allocation, and thus prevent default in the appropriate states.

For each country $i = 1, 2$ in the union, let $a_i(\bar{y}'; b_i, \bar{b}, \bar{y})$ denote the net stock of the Arrow security that pays in state $\bar{y}'$ tomorrow, conditional on individual wealth b_i and the aggregate state $(\bar{b}, \bar{y})$, with price $q(\bar{y}'; \bar{b}, \bar{y})$. Let $b_i'(b_i, \bar{b}, \bar{y})$ denote the net stock of foreign bonds that earn interest r tomorrow.

Let also $\tau(\bar{b}, \bar{y})$ denote the subsidy rate on net asset purchases, and $T_i(b_i, \bar{b}, \bar{y})$ the lump-sum income tax faced by country i.

In a competitive equilibrium with capital controls, country i solves the following problem for every current state

$$V_i(b_i, \bar{b}, \bar{y}) = \max_{c_i, b_i', \{a_i(\bar{y}')\}} \left\{ u\left(c_i\right) + \beta \sum_{\bar{y}'} \pi\left(\bar{y}'|\bar{y}\right) V_i\left(b_i', \bar{b}, \bar{y}'\right) \right\}$$

subject to

$$c_i + \left(1 - \tau\left(\bar{b}, \bar{y}\right)\right) \left(b_i' + \sum_{\bar{y}'} q\left(\bar{y}'; \bar{b}, \bar{y}\right) a_i\left(\bar{y}'\right) \right) \quad = \quad b_i + T_i\left(b_i, \bar{b}, \bar{y}\right) \tag{B.1}$$

and to a perceived law of motion for aggregate foreign asset holding $\bar{b}$.

The government is assumed to run a balanced budget for each country separately, that is

$$\tau\left(\bar{b}, \bar{y}\right) \left(b_i'\left(b_i, \bar{b}, \bar{y}\right) + \sum_{\bar{y}'} q\left(\bar{y}'; \bar{b}, \bar{y}\right) a_i\left(\bar{y}'; b_i, \bar{b}, \bar{y}\right) \right) = T_i\left(b_i, \bar{b}, \bar{y}\right) \tag{B.2}$$

for every current state and for each i.

A competitive equilibrium with tax subsidies is defined in the standard way, as (i) optimal decision rules that solve each country's problem given prices, government policy, and a perceived

[21] These authors consider taxes on borrowing instead of saving subsidies, although the two are equivalent. Wright (2006) also studies an alternative decentralization based upon country-specific borrowing limits, along the lines of Alvarez and Jermann (2000).

law of motion for aggregate wealth; (ii) a government policy that satisfies the balanced budget constraints given prices and individual decisions; (iii) Arrow security prices that clear asset markets; and (iv) consistency between the perceived law of motion for aggregate asset holding and the individual decision rules.

Our goal here is to show that there exists a government tax and transfer policy that supports the constrained-efficient allocation as a competitive equilibrium. We focus on the key steps of the argument.

Consider the first-order conditions to the country's problem

$$1-\tau\left(\bar{b},\bar{y}\right) = (1+r)\sum_{y'}\pi\left(\bar{y}'|\bar{y}\right)\frac{\beta u'\left(c_i(b_i',\bar{b}',\bar{y}')\right)}{u'\left(c_i(b_i,\bar{b},\bar{y})\right)} \tag{B.3}$$

$$\left(1-\tau\left(\bar{b},\bar{y}\right)\right)q\left(\bar{y}';\bar{b},\bar{y}\right) = \pi\left(\bar{y}'|\bar{y}\right)\frac{\beta u'\left(c_i\left(b_i',\bar{b}',\bar{y}'\right)\right)}{u'\left(c_i\left(b_i,\bar{b},\bar{y}\right)\right)}. \tag{B.4}$$

Given isoelastic preferences, the last equation implies

$$\frac{c_i\left(b_i',\bar{b}',\bar{y}'\right)}{c_i\left(b_i,\bar{b},\bar{y}\right)} = \frac{c\left(\bar{b}',\bar{y}'\right)}{c\left(\bar{b},\bar{y}\right)} \quad \text{for } i=1,2. \tag{B.5}$$

The two Euler equations imply

$$1=(1+r)\sum_{\bar{y}'}q\left(\bar{y}';\bar{b},\bar{y}\right). \tag{B.6}$$

Note also that, at the optimum, we may use (B.2) to eliminate subsidies and transfers from (B.1):

$$c_i\left(b_i,\bar{b},\bar{y}\right)+b_i'\left(b_i,\bar{b},\bar{y}\right)+\sum_{\bar{y}'}q\left(\bar{y}';\bar{b},\bar{y}\right)a_i\left(\bar{y}';b_i,\bar{b},\bar{y}\right)=b_i. \tag{B.7}$$

Consider now the constrained-efficient allocation, the solution to problem (P1′). This allocation, which we denote with a star superscript, satisfies the planner's Euler equation

$$u'\left(c^*\left(\bar{b},\bar{y}\right)\right)-\phi^*\left(\bar{b},\bar{y}\right)=\beta(1+r)\sum_{\bar{y}'}\pi(\bar{y}'|\bar{y})u'\left(c^*\left(\bar{b}',\bar{y}'\right)\right). \tag{B.8}$$

Using (B.5) in (B.3), and requiring that the resulting allocation be consistent with (B.8), it is easy to compute the state-contingent subsidy rates that implement the constrained-optimal allocation as

$$\tau\left(\bar{b},\bar{y}\right)=\frac{\phi^*\left(\bar{b},\bar{y}\right)}{u'\left(c^*\left(\bar{b},\bar{y}\right)\right)}. \tag{B.9}$$

Note that if the borrowing constraint to problem (P1′) does not bind in state $\left(\bar{b},\bar{y}\right)$, then $\phi^*\left(\bar{b},\bar{y}\right)=0$ and so $\tau\left(\bar{b},\bar{y}\right)=0$. In this case, from (B.4) and (B.6), the domestic interest rate equals the world interest rate. If the constraint is instead binding, then the (post-subsidy) domestic interest rate is higher than the world interest rate. This ensures that countries save in a constrained-optimal way, and that equilibrium borrowing is self-enforcing.

It is relatively straightforward to show formally that, given a constrained-efficient allocation that solves (P1′) and (P2) for the appropriate set of welfare weights, one can obtain individual asset holdings from (B.7) together with the market clearing condition for Arrow securities, Arrow security prices from (B.4), and a government policy from (B.9) and (B.2) that support that allocation as a competitive equilibrium with tax subsidies.

To find the appropriate set of welfare weights, we use the method proposed by Negishi (1960). This method exploits the equivalence between the market and the constrained-efficient allocations.

We obtain the time-0 present value budget constraint of country i by iterating forward on the flow budget constraint (B.7). We express it as

$$C_i\left(b_{i0}, \bar{b}_0, \bar{y}_0\right) = Y_i\left(\bar{b}_0, \bar{y}_0\right) + (1+r)\, b_{i0},$$

where $C_i\left(b_{i0}, \bar{b}_0, \bar{y}_0\right)$ and $Y_i\left(\bar{b}_0, \bar{y}_0\right)$ are the time-0 present-values of consumption and the endowment, respectively. At time 0, the time of forming the union, $\bar{y}_0$ is the union's endowment pair, b_{i0} is country i's net stock of foreign bonds, and $\bar{b}_0 = \sum_i b_{i0} = \sum_i y_{i0} + (1+r)\sum_i b_{i0}$ is the union's aggregate wealth.

It follows from (4.4) that we may express the present value of individual consumption as fraction of the present value of aggregate (constrained-efficient) consumption, that is $C_i\left(b_i, \bar{b}, \bar{y}\right) = \alpha_i C^*\left(\bar{b}, \bar{y}\right)$. Replacing above allows us to recover the individual consumption share parameters as

$$\alpha_i = \frac{(1+r)\, b_{i0} + Y_i\left(\bar{b}_0, \bar{y}_0\right)}{C^*\left(\bar{b}, \bar{y}\right)}. \tag{B.10}$$

Given equilibrium Arrow security prices $q\left(\bar{y}'; \bar{b}, \bar{y}\right)$, and optimal decision rules $c^*\left(\bar{b}, \bar{y}\right)$ and $b^{*\prime}\left(\bar{b}, \bar{y}\right)$, the C^* and Y functions solve the following functional equations

$$Y_i\left(\bar{b}, \bar{y}\right) = y_i + \sum_{\bar{y}'} q\left(\bar{y}'; \bar{b}, \bar{y}\right) Y_i\left(\bar{b}', \bar{y}'\right) \tag{B.11}$$

$$C^*\left(\bar{b}, \bar{y}\right) = c^*\left(\bar{b}, \bar{y}\right) + \sum_{\bar{y}'} q\left(\bar{y}'; \bar{b}, \bar{y}\right) C^*\left(\bar{b}', \bar{y}'\right) \tag{B.12}$$

with

$$\bar{b}' = \sum_i y_i' + (1+r)\, b^{*\prime}\left(\bar{b}, \bar{y}\right).$$

Notice that although it is straightforward to obtain the welfare weights from the consumption share parameters, we only need to know the α_i's in order to uncover the individual allocations.

C Numerical algorithms

C.1 World economy equilibrium

Our algorithm can be described in the following steps:

1. Solve for the autarky value function $V_{aut}(y)$ from equation (3.6).

2. Given a current guess for the equilibrium interest rate r, solve problem (P0) by iterating on the following steps:

   (a) Consider the n^{th} iteration, with a current conjecture for the debt limit b_n^W. For the initial conjecture, we use the natural borrowing constraint.

   (b) Given b_n^W, solve problem (P0) by policy function iteration. We discretize the state-space and use cubic-spline interpolation to compute decisions outside the grid.

      i. First find the decision rules that solve the system of first-order conditions to problem (P0), ignoring the debt limit. Consider the j^{th} iteration, with a current conjecture for the consumption decision rule $c_n^j(b,y)$. Compute a candidate update $c_n^{j+1}(b,y)$ by solving

$$u'\left(c_n^{j+1}(b,y)\right) = \beta(1+r)\sum_{y'} \pi(y'|y)u'\left(c_n^j(b',y')\right)$$

      with

$$b' = y + (1+r)b - c_n^{j+1}(b,y).$$

      As part of the solution, we obtain $b_n'^{j+1}(b,y)$.

      ii. Check whether the borrowing constraint is violated. If $b_n'^{j+1}(b,y) < b_n^W$, then update the solution as follows:

$$\begin{aligned}
b_n'^{j+1}(b,y) &= b_n^W \\
c_n^{j+1}(b,y) &= b - b_n'^{j+1}(b,y) \\
\phi_n^{j+1}(b,y) &= u'\left(c_n^{j+1}(b,y)\right) - \beta(1+r)\sum_{y'} \pi(y'|y)u'\left(c_n^{j+1}(b',y')\right),
\end{aligned}$$

      If instead $b_n'^{j+1}(b,y) \geq b_n^W$, then update using the unconstrained solution, setting also $\phi_n^{j+1}(b,y) = 0$.

      iii. Iterate on the previous two steps until the decision rules converge. At the end, compute the value function $V_n(b,y)$.

(c) Given $V_n(b, y)$, update the debt limit as follows:

$$b^W_{n+1} = \max_{y'} \left\{ b_{y'} : V_n(b_{y'}, y') = V_{aut}(y') \right\}.$$

(d) Iterate on steps 2b and 2c until the borrowing limits converge.

3. Check the market clearing condition by approximating the aggregate bond holding in the world economy with the total bond holding of a particular country over a very long simulation period. We discretize the state-space using a finer grid, and linearly interpolate the decision rules.

4. Iterate on steps 2 and 3 until we find an interest rate that approximately clears the bond market.

C.2 Union problem under centralized default

Our algorithm to solve for the union's allocation given an equilibrium world interest rate r can be described as follows:

1. Solve problem (P1$'$) using the method described in step 2 of the algorithm of Section C.1. As part of the solution we obtain the union decision rule $c^*(\bar{b}, \bar{y})$, the multiplier function $\phi^*(\bar{b}, \bar{y})$, and the value function $V^U(\bar{b}, \bar{y})$.

2. Decentralize the union's constrained-efficient allocation as a competitive equilibrium with capital controls.

   (a) Compute tax-subsidies from (B.9).

   (b) Compute pre-subsidy Arrow-security prices from (B.4).

   (c) Compute the present-value functions from (B.11) and (B.12). In practice, we guess some arbitrary functions on a grid and then iterate on the two recursive equations until convergence. We linearly interpolate these functions when future wealth levels fall outside the grid.

   (d) Compute consumption shares from (B.10).

   (e) Compute the value function for each country from (4.6).

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